Reteach

Finding Real Roots of Polynomial Equations

To find the roots of a polynomial equation, set the equation equal to zero. Factor the polynomial expression completely. Then set each factor equal to zero to solve for the variable.

Solve the equation: \(2x^5 + 6x^4 = 8x^3\).

**Step 1** To set the equation equal to 0, rearrange the equation so that all the terms are on one side.

\[2x^5 + 6x^4 = 8x^3\]
\[2x^5 + 6x^4 - 8x^3 = 0\]

**Step 2** Look for the greatest number and the greatest power of \(x\) that can be factored from each term.

\[2x^5 + 6x^4 - 8x^3 = 0\]
\[2x^3(x^2 + 3x - 4) = 0\]

**Step 3** Factor the quadratic.

\[2x^3(x^2 + 3x - 4) = 0\]
\[2x^3(x + 4)(x - 1) = 0\]

**Step 4** Set each factor equal to 0.

\[2x^3 = 0 \quad x + 4 = 0 \quad x - 1 = 0\]

**Step 5** Solve each equation.

\[x = 0 \quad x = -4 \quad x = 1\]

The solutions of the equation are called the roots.

The roots are \(-4, 0,\) and \(1\).

**Solve each polynomial equation.**

1. \(3x^6 - 9x^5 = 30x^4\)
   \[3x^6 - 9x^5 - 30x^4 = 0\]
   \[3x^4(x^2 - 3x - 10) = 0\]

2. \(x^4 + 6x^2 = 5x^3\)
   \[x^4 - 5x^3 + 6x^2 = 0\]

3. \(2x^3 - 6x^2 - 36x = 0\)

4. \(2x^6 - 32x^4 = 0\)
You can use the Rational Root Theorem to find rational roots.

**Rational Root Theorem**

If a polynomial has integer coefficients, then every rational root can be written in the form \( \frac{p}{q} \), where \( p \) is a factor of the constant term and \( q \) is a factor of the leading coefficient.

Use the Rational Root Theorem. Solve the equation: \( x^3 + 3x^2 - 6x - 8 = 0 \).

The constant term is \(-8\). The leading coefficient is 1.

- \( p \): factors of \(-8\) are ±1, ±2, ±4, ±8
- \( q \): factors of 1 are ±1

Possible roots, \( \frac{p}{q} \): ±1, ±2, ±4, ±8

Try:

Find the remaining rational roots

Roots are:

Use the Rational Root Theorem. Solve \( x^3 - 7x^2 + 7x + 15 = 0 \).

5. a. Identify possible roots. 

   b. Use the synthetic division to identify an actual root.

   c. Solve the remaining quadratic to identify the remaining roots.

   d. Roots are:
Reteach

**Fundamental Theorem of Algebra (continued)**

To solve \(x^4 + x^3 - 5x^2 + x - 6 = 0\) means to find all the roots of the equation. A fourth degree equation has 4 roots.

**Step 1** Identify possible real roots.
Possible roots, \(\frac{p}{q}\): \(\pm 1, \pm 2, \pm 3, \pm 6\)

**Step 2** Test 2 as a root using synthetic substitution.

\[
\begin{array}{c|cccc}
2 & 1 & 1 & -5 & 1 & -6 \\
& & 2 & 6 & 2 & 6 \\
\hline
1 & 3 & 1 & 3 & 0
\end{array}
\]

The remainder is 0, so 2 is a root. \((x - 2)(x^3 + 3x^2 + x + 3) = 0\)

Test -3 as a root using synthetic substitution.

\[
\begin{array}{c|cccc}
-3 & 1 & 3 & 1 & 3 \\
& & -3 & 0 & -3 \\
\hline
1 & 0 & 1 & 0
\end{array}
\]

The remainder is 0, so -3 is a root. \((x - 2)(x + 3)(x^2 + 1) = 0\)

**Step 3** Find the remaining roots.
\[x^2 + 1 = 0\]
\[x = \pm i\]
The roots of the equation are 2, -3, i, and -i.

**Find the roots of the equation** \(x^4 - 3x^3 + 6x^2 - 12x + 8 = 0\).

1. a) Possible roots ______________________

   b) Use synthetic division to find a root ____________________

   c) Factor the remaining polynomial OR do synthetic division again to find another root. ____________________
d) If you used synthetic division a second time, solve the remaining polynomial.

e) All roots of the equation are _____________________________

Here are some more to practice on:

1. $2x^3 + 3x^2 - 8x - 12 = 0$   
2. $-3x^3 + 30x^2 + 5x - 50 = 0$   
3. $x^3 + 15x^2 + 75x + 125 = 0$

4. $x^3 - 2x^2 - 32x + 96 = 0$   
5. $8x^3 - 12x^2 + 6x - 1 = 0$   
6. $4x^3 + 16x^2 - 25x - 100 = 0$

7. $2x^4 - x^3 - 14x^2 - 5x + 6 = 0$   
8. $6x^3 - 11x^2 - 19x - 6 = 0$

9. $x^4 - 5x^3 + 15x^2 - 45x + 54 = 0$   
10. $2x^4 + 5x^3 - 10x^2 + 10x + 8 = 0$